Spherical astronomy of stars and deep-sky objects

End goal: Be able to calculate the position of a star or DSO to decent precision from its catalog coordinates

Pre-requisites:

* Functional knowledge of algebra and trigonometry
* Basic understanding of astronomy

Not covered: Positions of planets and solar system objects (This is much more complicated)

Outline:

Introductory topics (light on math):

- Units for angle measurement
- Calendars and time measurement
- Celestial sphere, celestial poles, celestial equator, horizon
- Horizontal coordinate system
- Airmass and atmospheric refraction
- Ecliptic and the equinoxes
- Equatorial coordinate system
- Hour angle and Sidereal time

Outline:

Main topics (more math-heavy)

- Julian Day and Calculation of Local Sidereal Time
- Spherical triangles and their solution
- Example application: North angle at a given alt/azimuth.
- Main application: Equatorial $\rightarrow$ horizontal coordinates
- Precession of the equinoxes
- Formulae for precession
- Brief discussion of other effects

Ref: Astronomical Algorithms Jean Meeus

Units for angle measurement in astronomy:

Degrees. — most common. Radians — for calculations.

1 arcminute $= \left(\frac{1}{60}\right)^\circ$ 1 arcsecond $= \left(\frac{1}{3600}\right)^\circ$

arcminute is denoted as $'$ and arcsecond as $''$

Eg: $15^\circ 30' 45'' = \left(15 + \frac{30}{60} + \frac{45}{3600}\right)^\circ = 15.5125^\circ$

"Hours": 1 hour $= 15^\circ$, used as a measure of angle

Based on the fact that earth rotates 360° in 24h.

Eg: $15^h 30^m 45^s = \left[\left(15 + \frac{30}{60} + \frac{45}{3600}\right) \times 15\right]^\circ$

$= 232.6875^\circ$

Quick Calculations: These approximate calculations help a lot for rough estimates.

Earth rotates 360° in 24 hours $\Rightarrow$ 15° in 1 hour,

$\Rightarrow$ $\boxed{1^\circ \text{ in } 4 \text{ minutes}}$

Earth revolves 360° in 365 days $\Rightarrow$ $\approx 1^\circ$ in a day

$\Rightarrow$ 30° (or 2 hours) in a month

$\boxed{1 \text{ month} \rightarrow 2 \text{ hours}}$

not sun/moon/planet

i.e. star that is "on the meridian" (approx: overhead) at 8PM on 1st Jan, will be $\approx 30^\circ$ west at 8PM on 1st Feb

Calendars:

The calendar we use today is called the Gregorian Calendar.

An important older calendar: Julian Calendar

$$\underbrace{1 \text{ year}}_{\text{cycle of seasons}} = 365.24219 \underbrace{\text{days}}_{\text{rotation}}$$

Julian calendar: Assumed 1 Julian year = 365.25 days.

Hence: Every year has 365 days except for Every 4^{th} year (leap year) which has 366 days $\Rightarrow$ Average year has 365.25 days

Gregorian Calendar: Brought two changes:

1) The average year is 365.2425 days long.
This is split as:

Normal year = 365 days
Every 4^{th} year except centuries = 366 days
Every 100^{th} year = 365 days
Every 400^{th} year = 366 days (eg: 2000)

2) After 4^{th} Oct 1582, the next day was 15^{th} Oct 1582.

Time Measurement:

We measure "mean solar time" for our activities.

i.e. Every day has the same length although sun has different rate of E-W motion as seen from earth depending on the season.

The earth's rotation speed varies

* Mean Solar Time tracks this variation (Leap seconds are added from time-to-time)
* "Dynamical time" is the real uniform time that is unaffected by this variation. (for ephemerides)

Timezones: UTC → World standard tracking (mean) solar transit over Greenwich

Your timezone has a UTC offset

India : UTC + 5:30

California : UTC − 8 in winter

UTC − 7 in summer

etc.

Astronomers tend to always measure time in UTC.

⇒ First step in any calculation is to convert Local Time (LT) to UTC (a.k.a. ≈ UT)

Local mean time at a specific place is

$$\text{UTC} + \frac{\text{longitude in degrees}}{15} \text{ hours.}$$

[Convention: Eastern longitudes are +ve]

Timezones are a convenience so we don't go crazy.

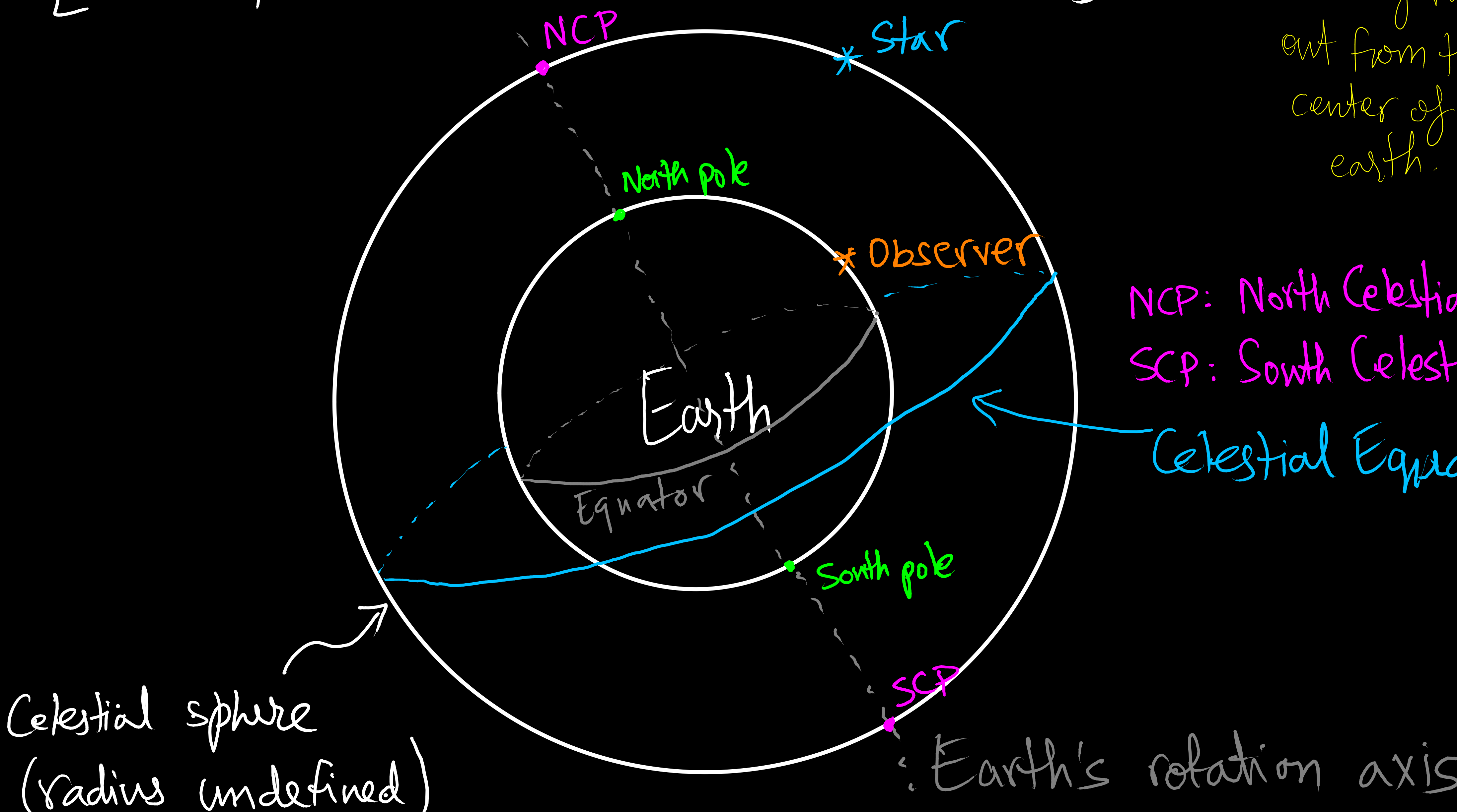
IST is calibrated to 82.5°E longitude

Eg: Local mean time for Bhuj city is

$\simeq$ IST − 50 min

The Celestial Sphere: All celestial objects appear to lie on a sphere called the Celestial Sphere. [Our eyes can't estimate the distance!]

If this is confusing, think in terms of drawing rays out from the center of the earth.



NCP: North Celestial Pole

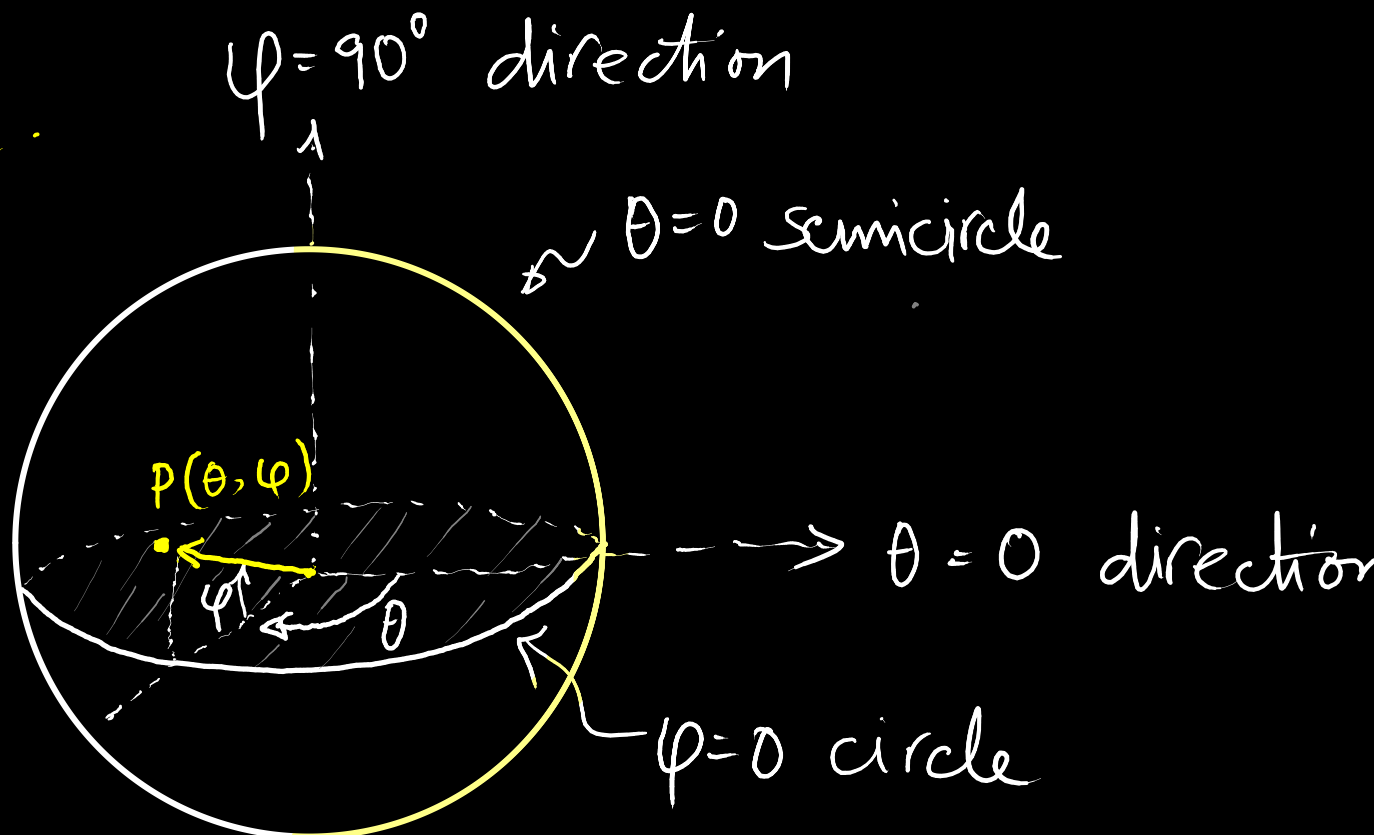
SCP: South Celestial Pole

Any point on a sphere can be described in terms of two angles:

Denoted by θ, φ here.

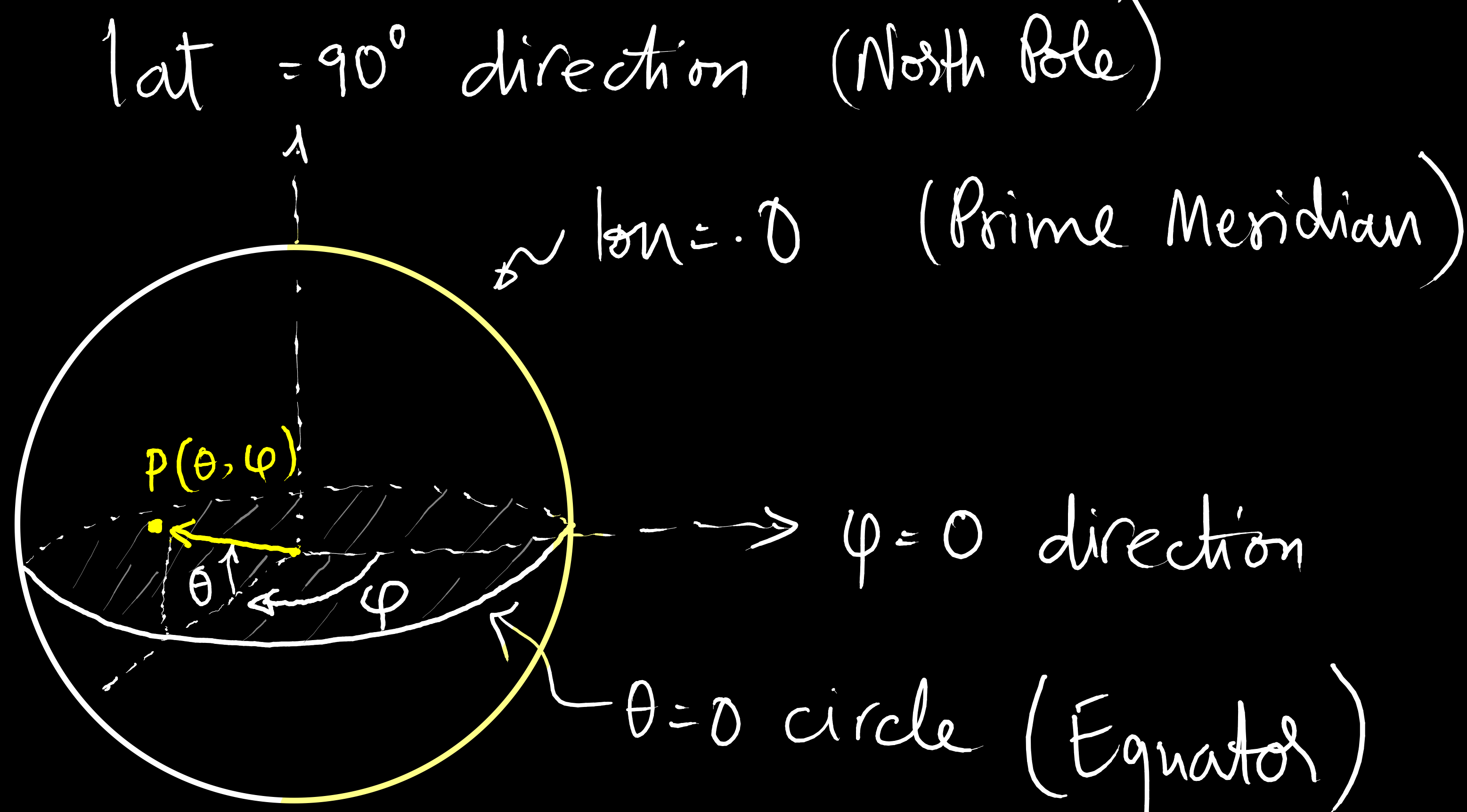
$-90^\circ \leq \varphi \leq 90^\circ$

$0^\circ \leq \theta < 360^\circ$



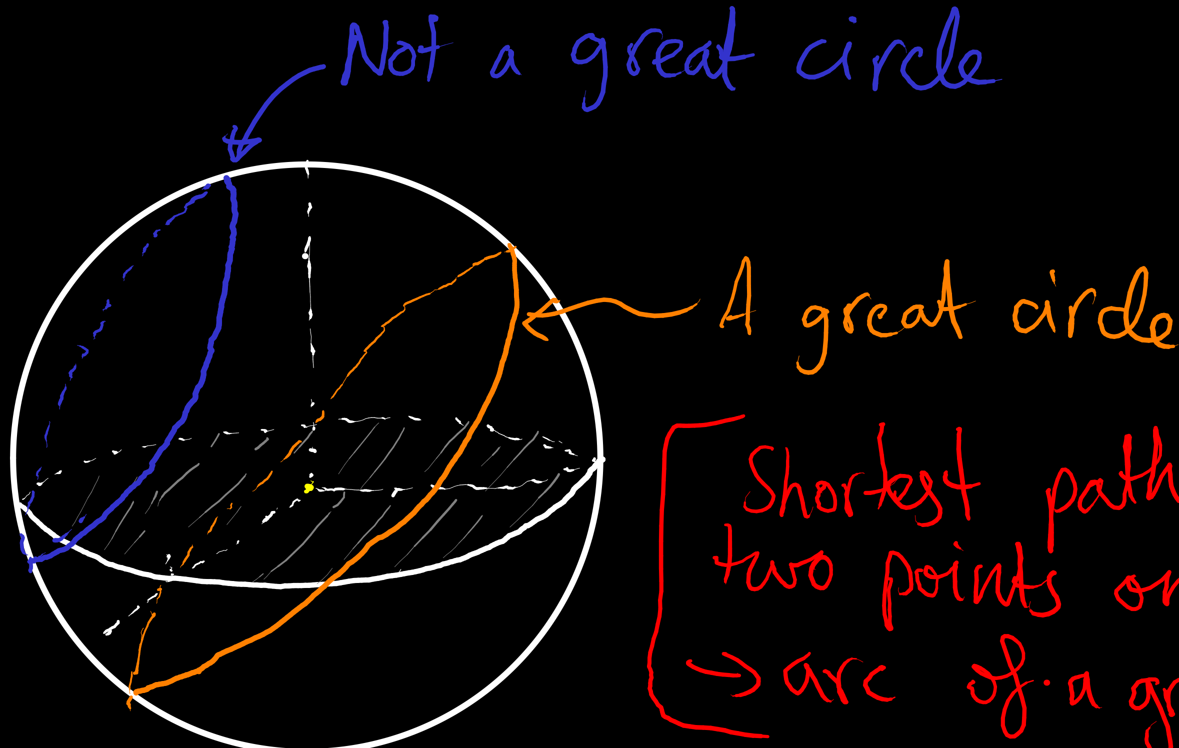
The coordinate system needs two reference directions / points / circles

For earth, we use latitude and longitude:



For the sky, we use RA and Declination (covered later)

Great circles: Circles on a sphere whose center coincides with the center of the sphere
(Radius of a great circle = radius of the sphere)



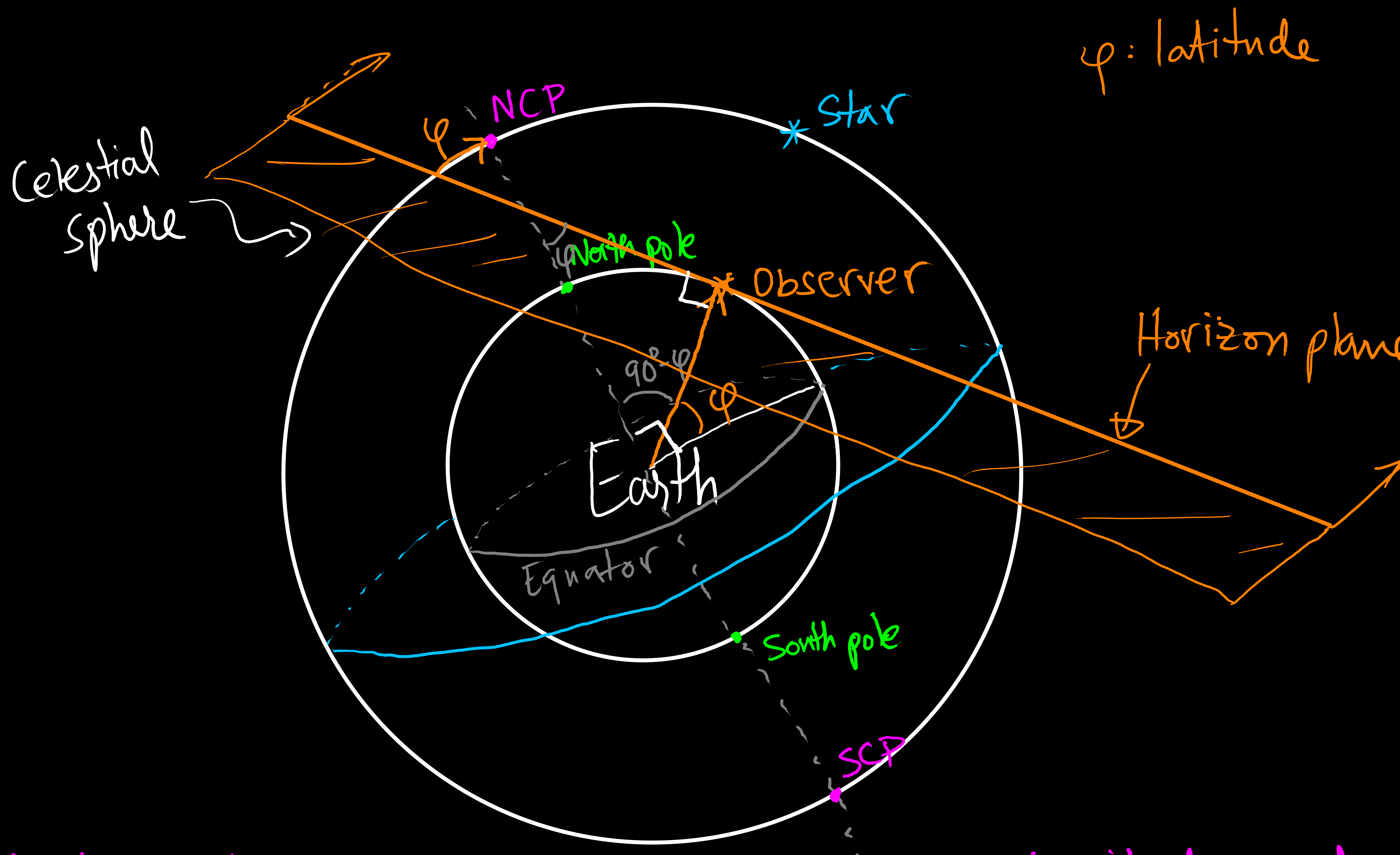
Shortest path between two points on a sphere → arc of a great circle

We say that the great circles are geodesics on the sphere

Equator is a great circle. Arctic circle, tropic of cancer etc are not. Circles of constant longitude are great semi-circles. The equator is the only circle of constant latitude that is a great circle

Horizon: is the plane tangent to the earth at the location of the observer

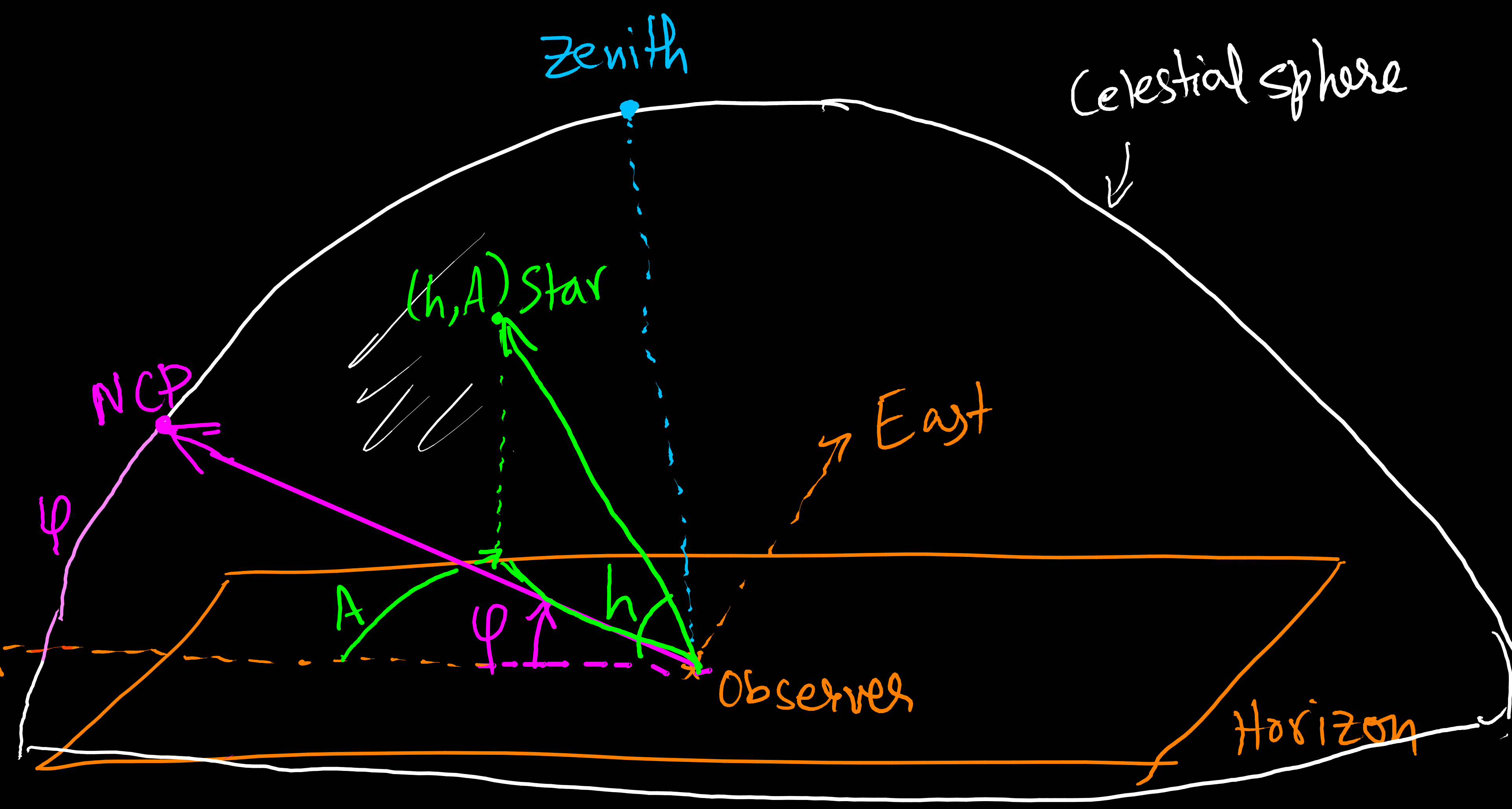
φ: latitude



Altitude of NCP as seen by Observer = Latitude of observer

Horizontal Coordinate System: The natural coordinate system for the observer.

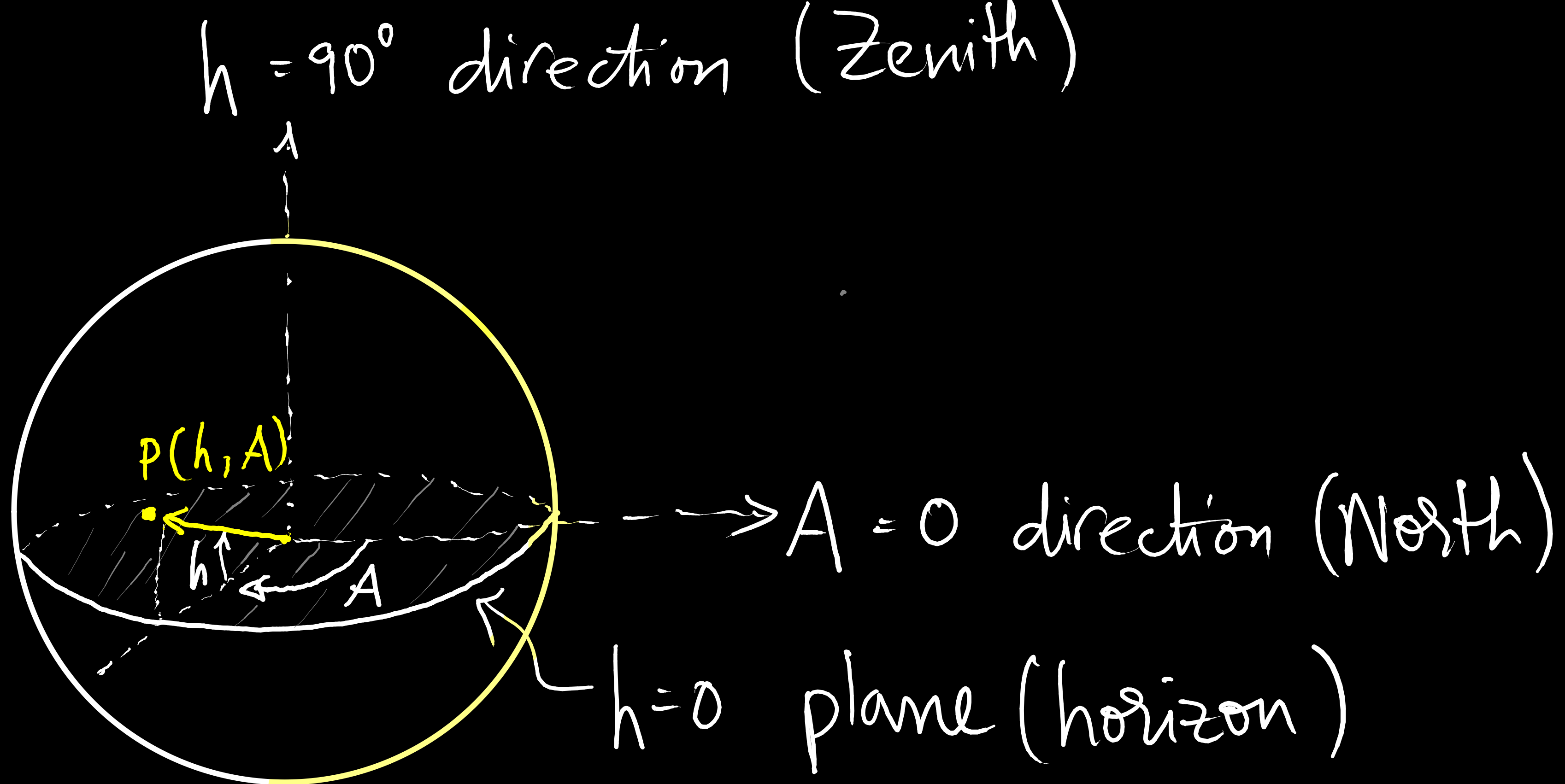
φ: latitude of observer



h : Altitude of the star

A : Azimuth of the star

Convention we will use: $A = 0$ at north and A increases eastwards

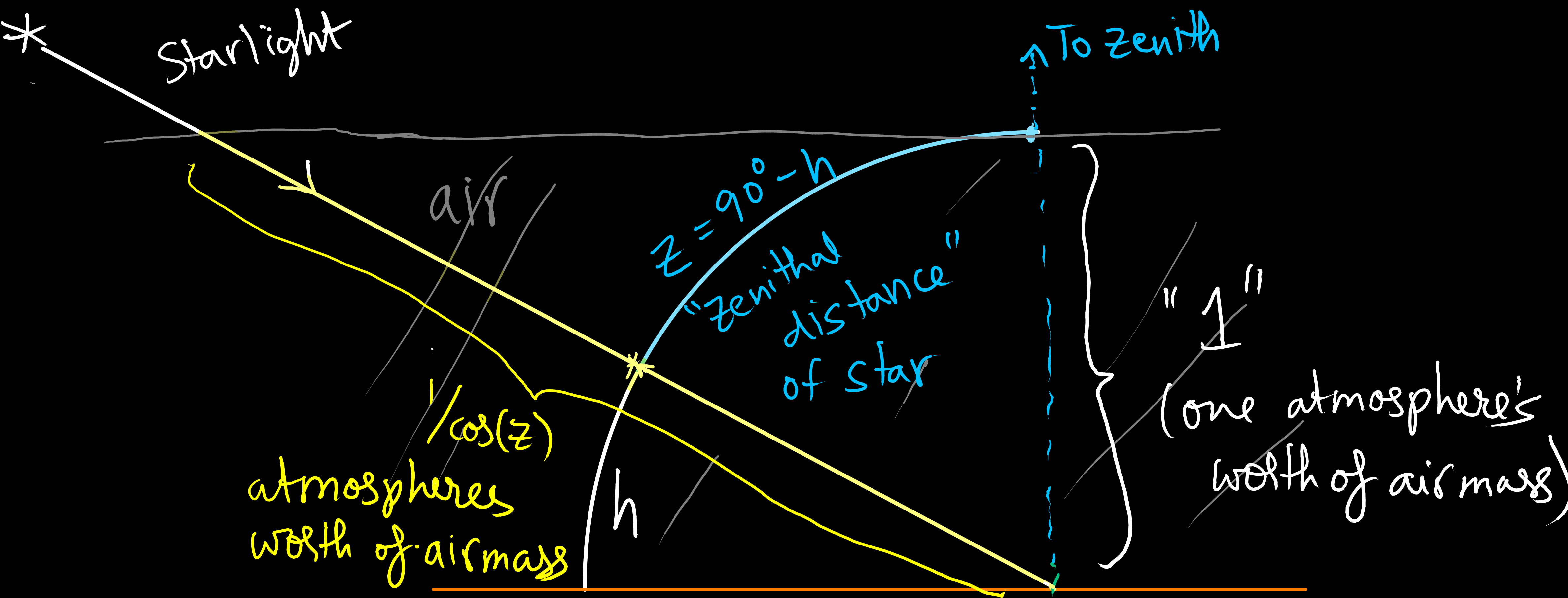


Altitude: Looking up and down

Azimuth: Looking around.

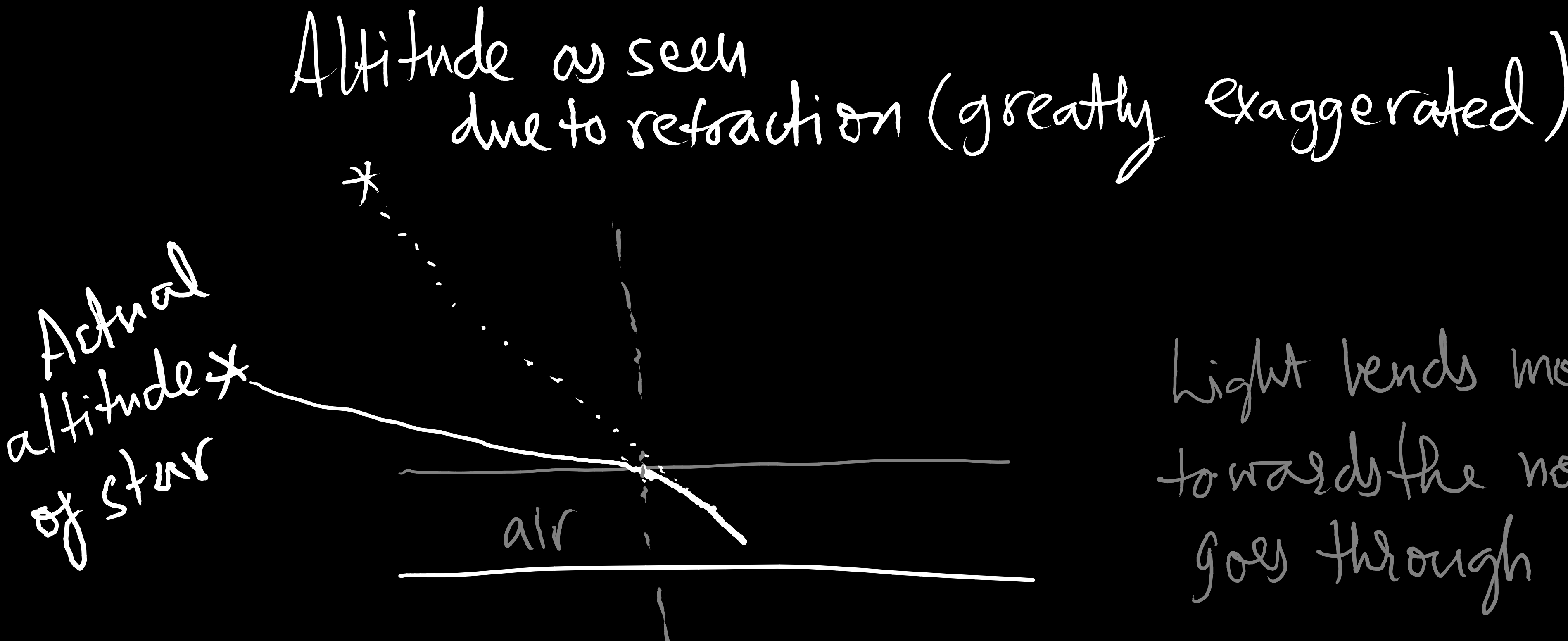
Airmass: Amount of atmosphere the starlight is going through

"Plane parallel approximation": Uniform, flat atmosphere



Airmass = $\sec(z)$

Atmospheric Refraction: Causes objects to appear higher than they actually are



Light bends more and more towards the normal as it goes through increasingly denser air

Atmospheric Refraction:

Empirical formula by Saemundsson

pressure

$$\text{correction} = \frac{p}{101\ \text{kPa}} \times \frac{283}{(273 + T/^\circ C)} \times \frac{1.02}{\tan\left(h + \frac{10.3}{h + 5.11}\right)} \text{ arcmin}$$

temperature

alt. in degrees

h must be in degrees and the argument to tan( ) should be interpreted as being in degrees. Correction is obtained in arcmin.

in degrees

in degrees

$\Rightarrow$ Apparent altitude $h_a = h + \dfrac{\text{correction}}{60}$

Atmospheric refraction: the inverse formula by G.G. Bennett

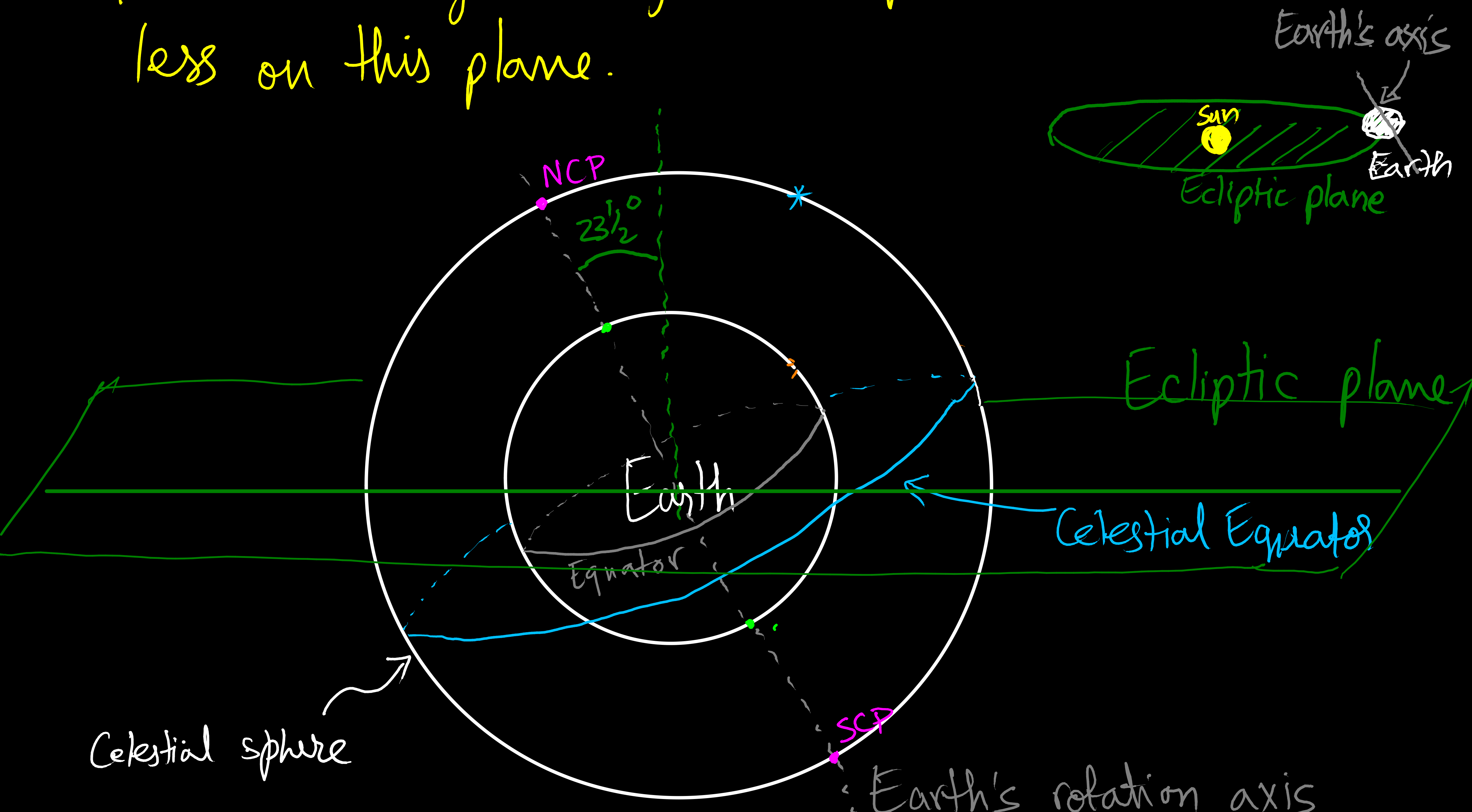
Note: the formulas are only approximately inverses of each other

$$\text{correction} = \frac{\overset{\text{pressure}}{p}}{101\ \text{kPa}} \times \frac{283}{(273 + \underset{\text{temperature}}{T}/{}^\circ C)} \times \frac{1}{\tan\left(\underset{\text{apparent alt. in degrees}}{h_a} + \frac{7.31}{h_a + 4.4}\right)} \ \text{arcmin}$$

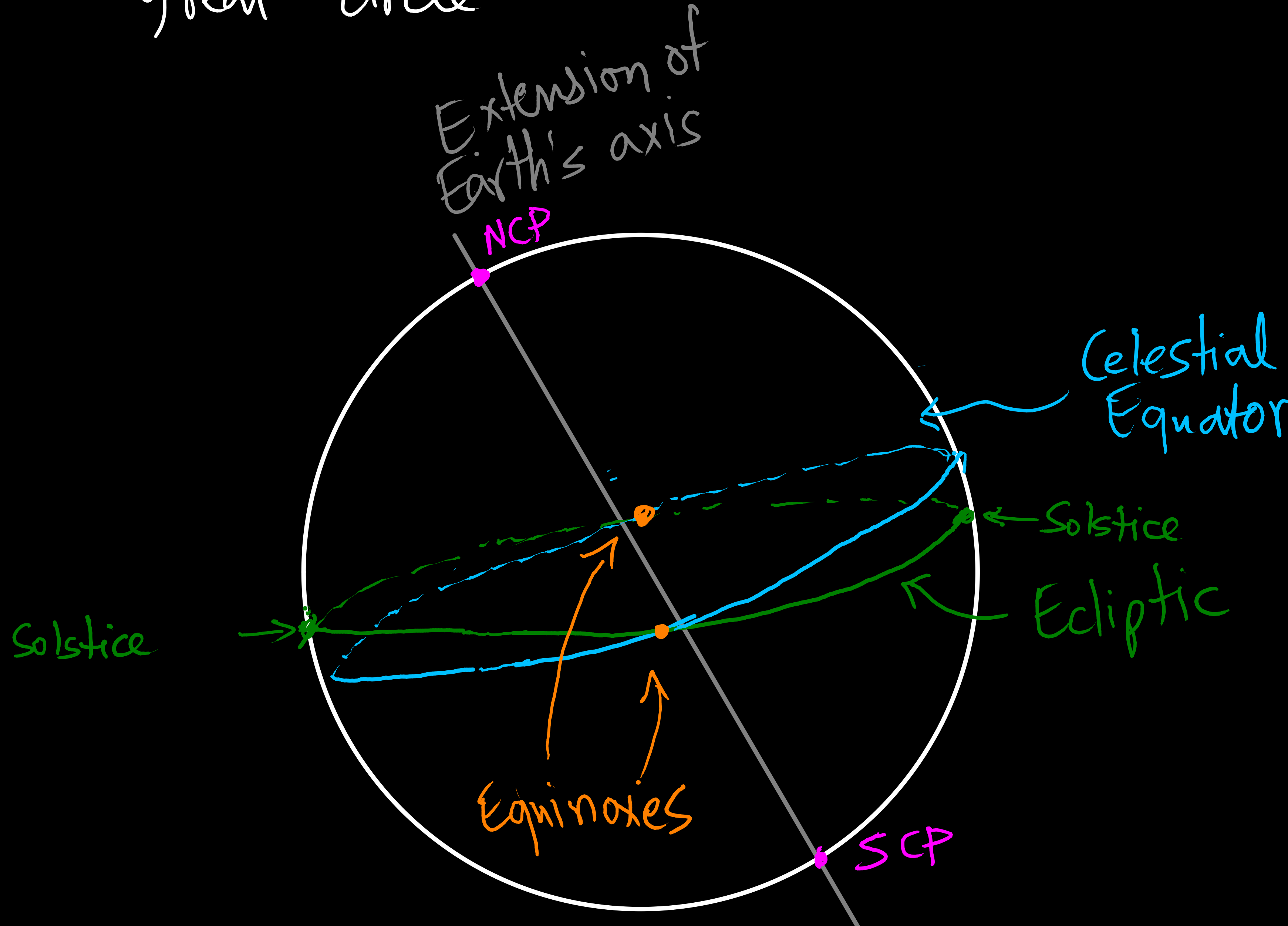
$$\overset{\text{degrees}}{h} = \overset{\text{degrees}}{h_a} - \frac{\text{correction}}{60}$$

Ecliptic: The plane of the earth's orbit around the sun.

Most solar system objects (except comets) lie more or less on this plane.



Ecliptic: When projected onto the celestial sphere, it is a great circle



The two circles intersect at two points called the equinoxes.

Equinoxes: The two equinoxes are called "Vernal" (Spring) Equinox and Autumnal Equinox.

When the sun is at these points, the day & night are of equal length.

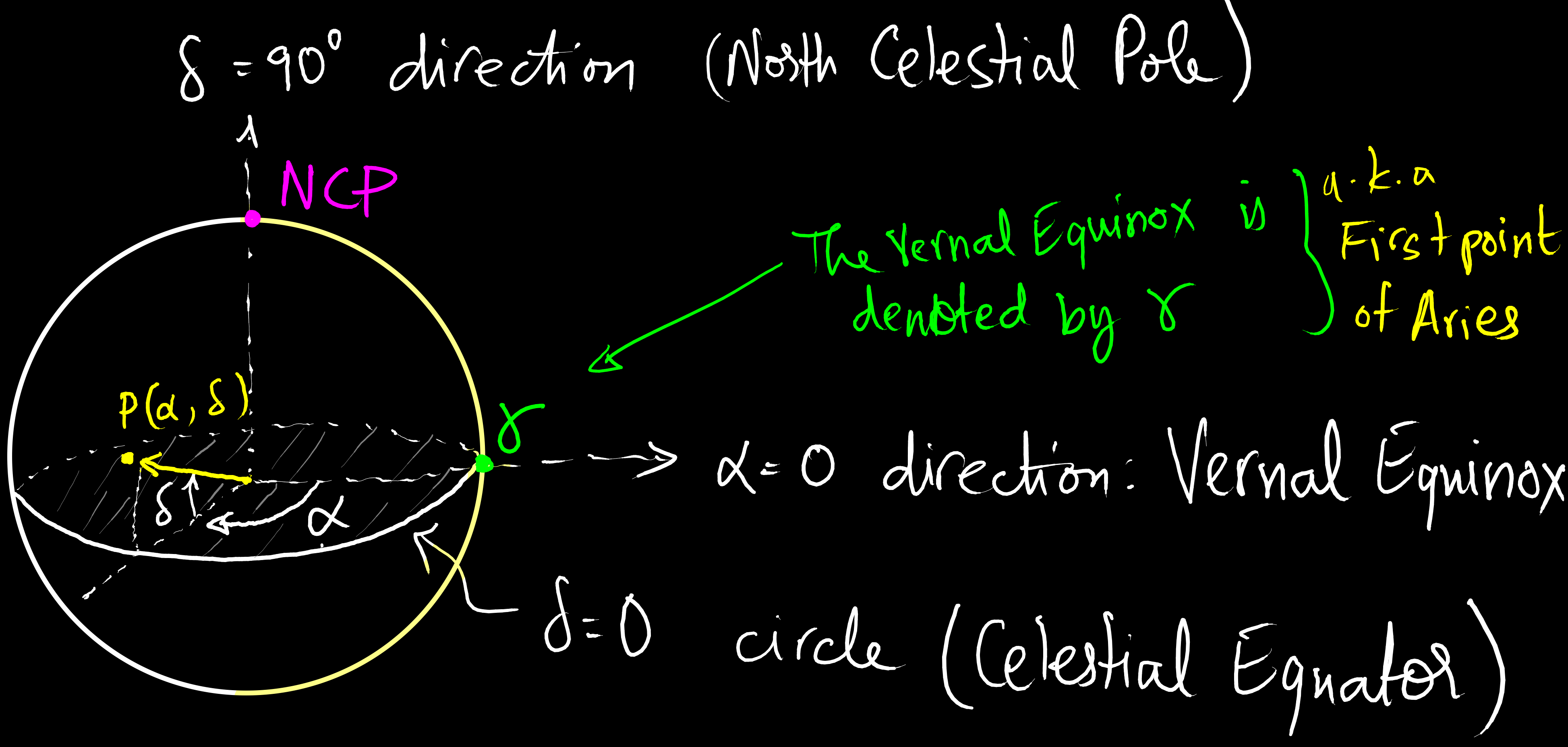
$\approx$ March 22^{nd} : Sun is at Vernal Equinox

$\approx$ September 22^{nd} : Sun is at Autumnal Equinox

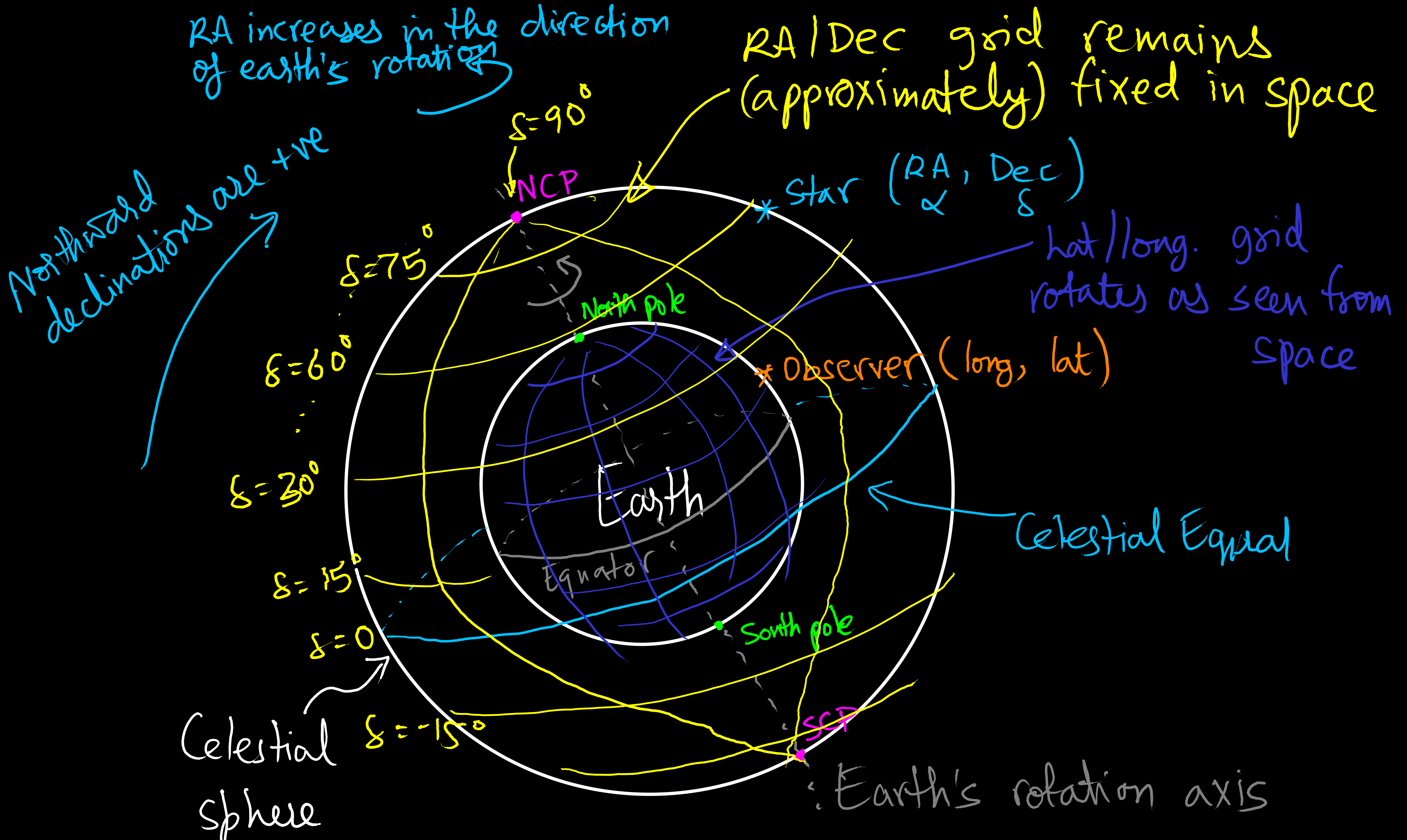
Equatorial Coordinates: A coordinate system based on the celestial equator and Vernal equinox that is (approximately) fixed to the sky.

α : Right Ascension (RA) – roughly like longitude

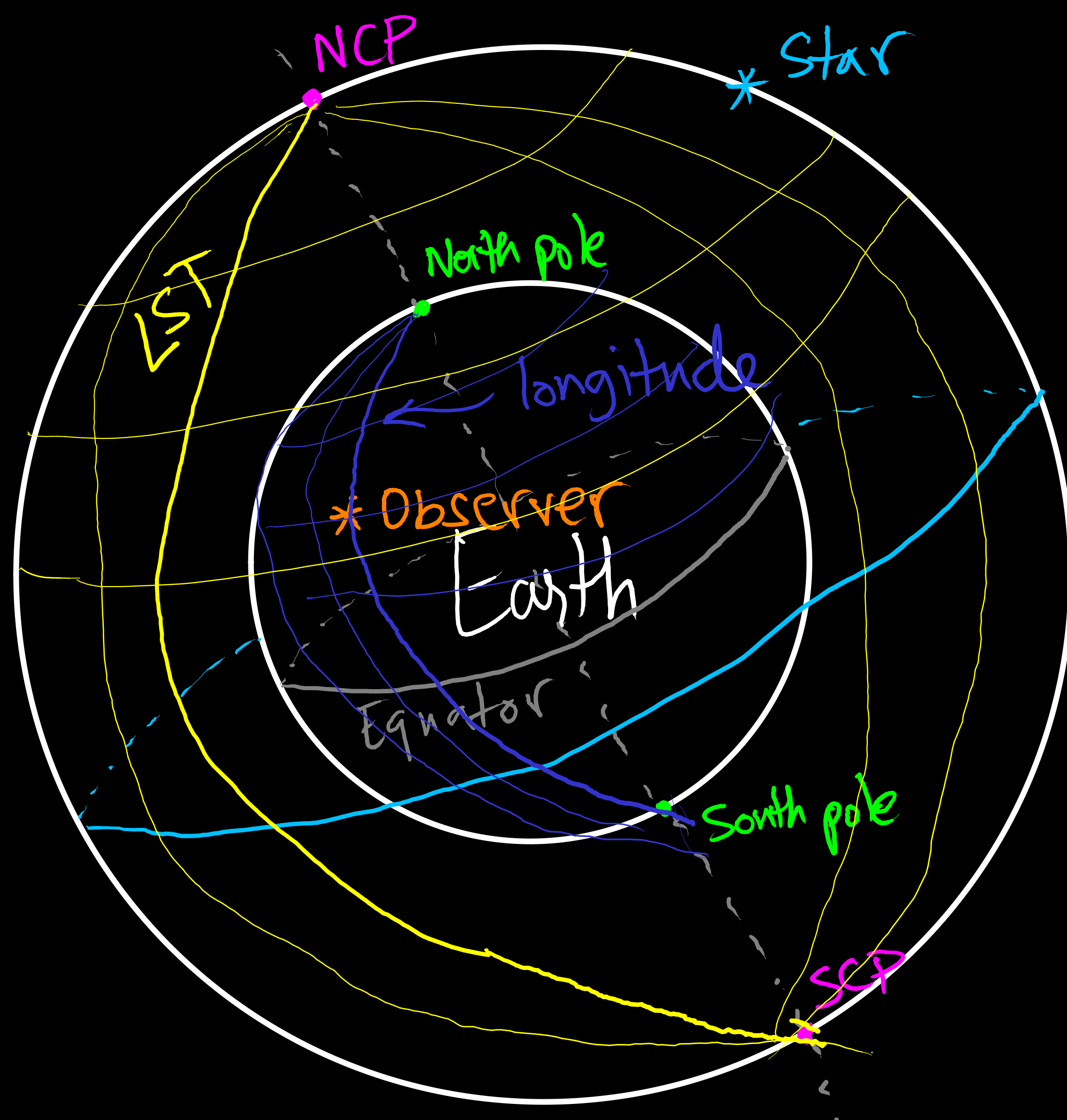
δ : Declination (Dec) – roughly like latitude.



Whereas the earth rotates,
latitudes and longitudes are fixed to the earth and rotate with it; RA and Dec are (approximately) fixed to the sky & do not rotate with the earth



Local Sidereal Time: The RA that coincides with the longitude of the observer (the "meridian") is the local sidereal time.



The Sidereal "Clock":

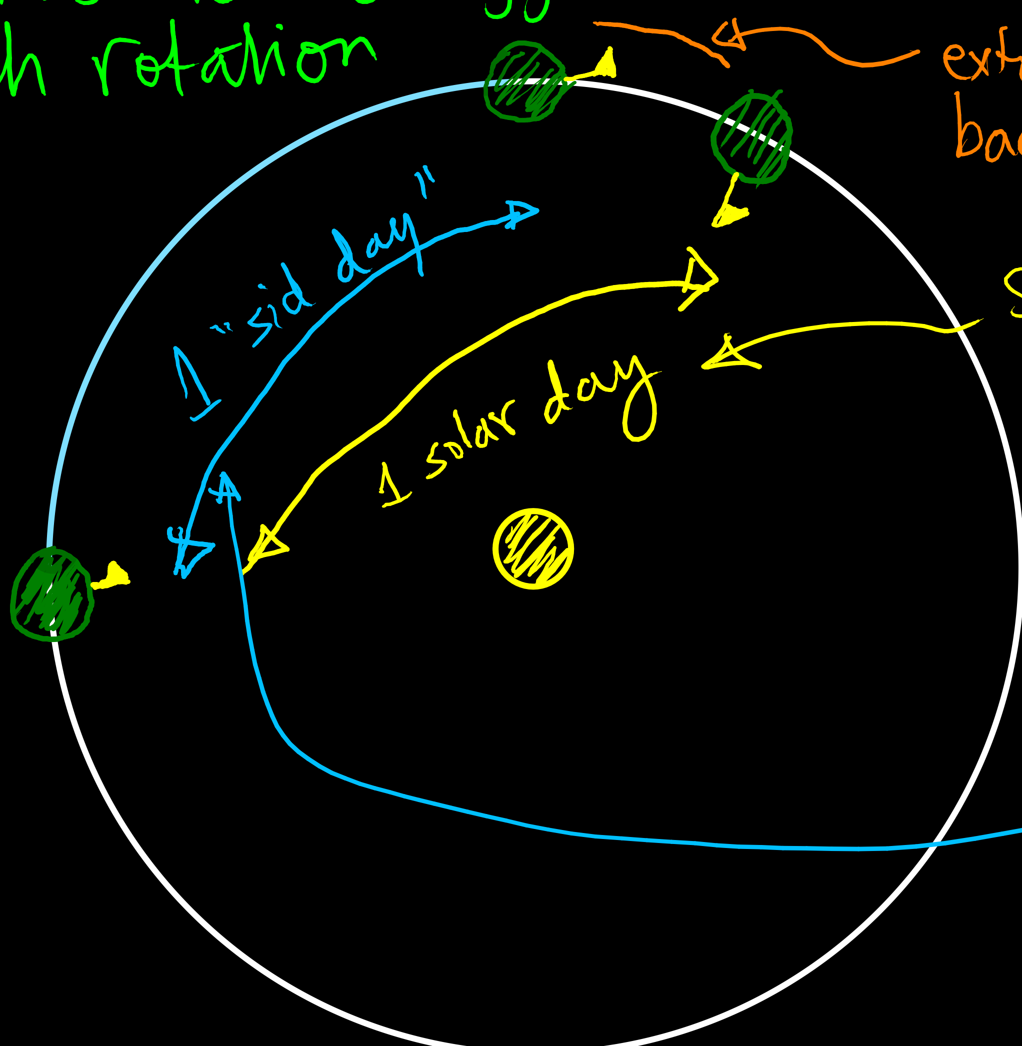
The sidereal clock follows the earth's position w.r.t. the stars*

(* ignoring precession)

The usual (mean solar) clock follows the earth's position w.r.t the sun.

To illustrate: Exaggerated example of a planet that revolves 90° for each rotation

extra rotation needed for the sun to come back to the same position



Sun comes back (roughly) to the same position

Distant Star *

360° rotation of planet as seen from space. Distant star comes back to same position

For earth: 1 (mean solar) day = 24 hours

rotation reqd to get star back in same position

$$\approx \underbrace{360^\circ}_{} + \underbrace{\frac{360^\circ}{365}}_{}$$

total rotation as seen from space

extra rotation needed to get the sun to the same spot

$$\approx 361^\circ$$

$\Rightarrow$ Sidereal day $= 24$ hours $- 1^\circ$

$= 24$ hours $- 4$ min

$$\Rightarrow \underline{23^h\,56^m}$$

More accurately: $23^h\,56^m\,4.0905^s$ or 86164.1 s.

1 second of time $\Rightarrow 1.0027379093\,5$ s change in LST.

Hour Angle:

$$HA = LST - \alpha$$

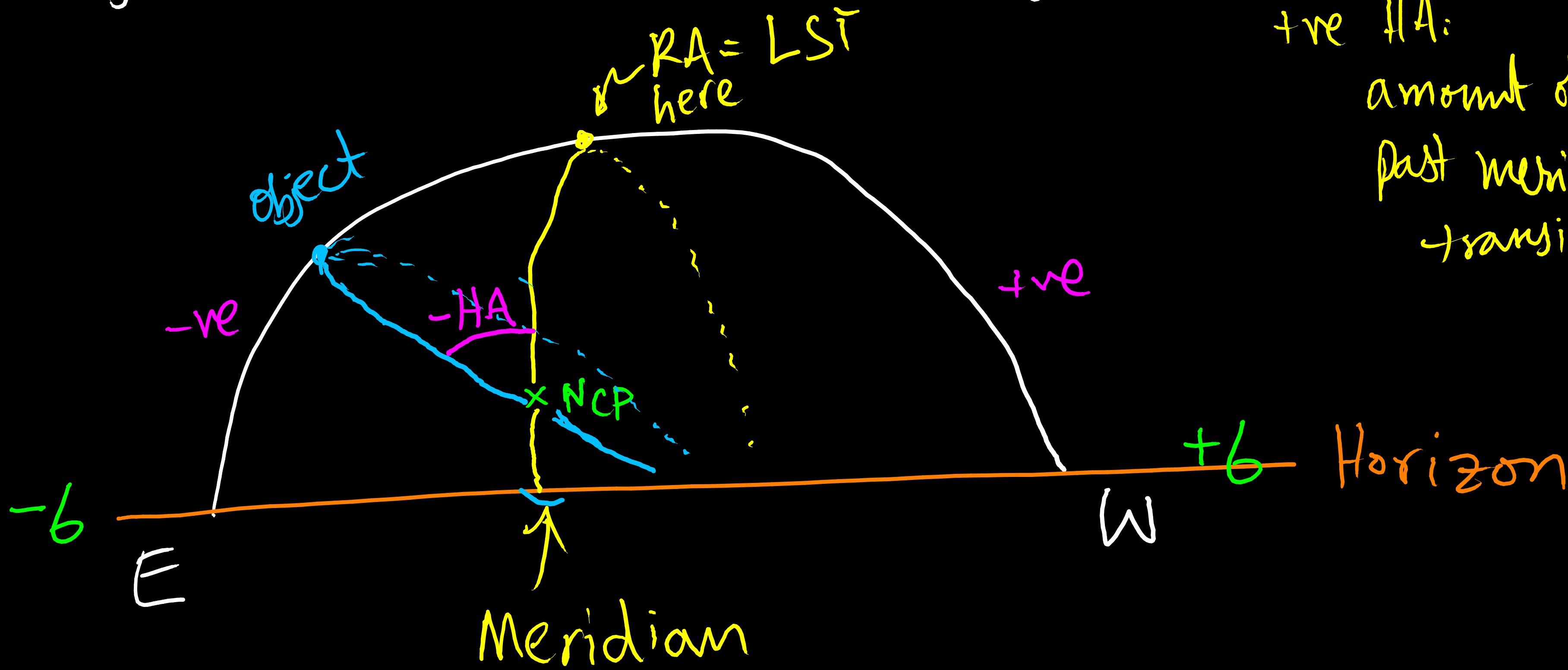
Hour angle of an object

RA at meridian

RA of object

-ve HA: amount of time object will get to meridian

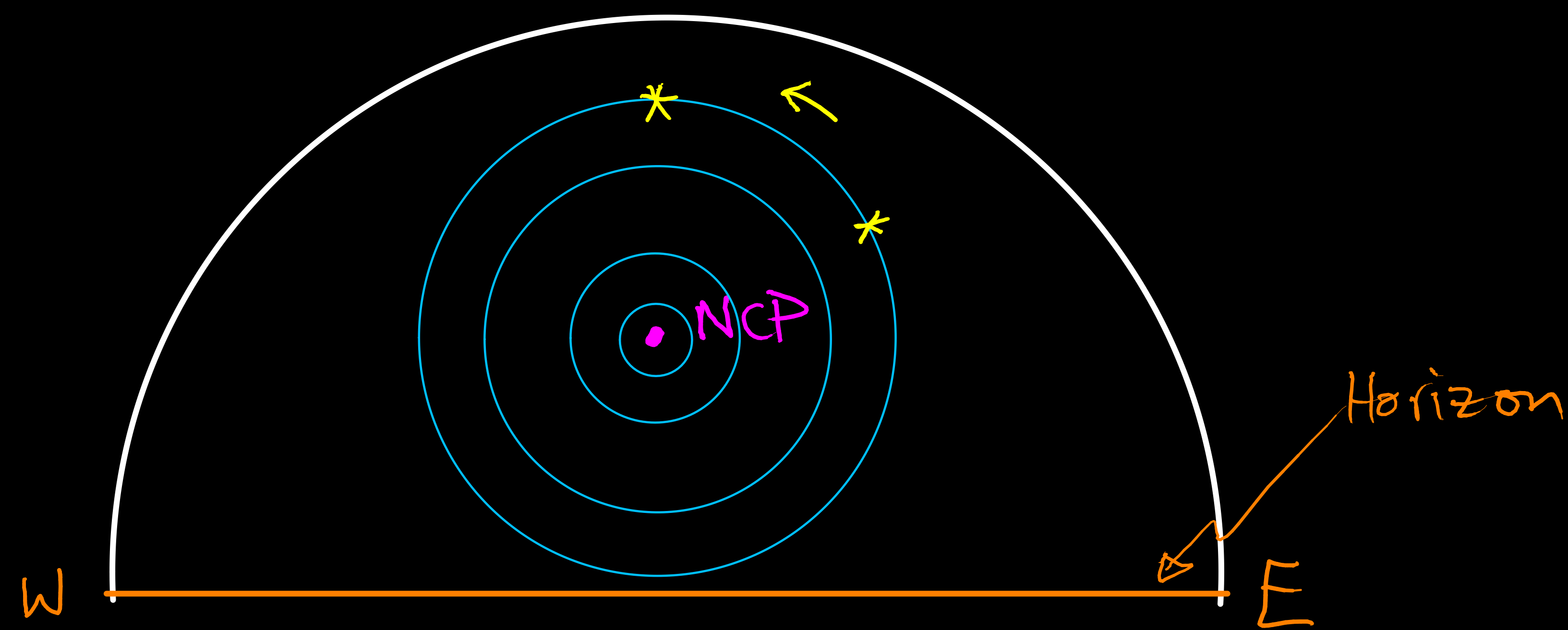
+ve HA: amount of time past meridian transit



Angle from the meridian (measured in hours); objects past the meridian have +ve values (west); before ⇒ -ve (east)

Motion of the stars in the sky:

The stars follow circles of fixed declination as the earth rotates.



The only great circle of fixed dec. is the celestial equator

Star on the equator moves $\simeq 1^\circ$ in 4 minutes.

Star at declination δ moves $\simeq 1^\circ \times \cos(\delta)$ in 4 minutes

(Although it's path still subtends 360° around the NCP, the angular distance it covers is only $360^\circ \times \cos(\delta)$)